The economics of parking occupancy sensors

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Abstract

Parking occupancy sensors are devices that assist search of vacant parking. The interplay between two government policies, installation of sensors and pricing parking, is studied. When a fraction of parking is equipped with sensors, it should be more expensive and be used by shorter-term parkers. Underpriced parking may dampen the incentive to install additional sensors, both absolute and relative to the incentive to build more parking. The convexity of welfare with respect to the number of sensors is ambiguous and depends on the pricing method.

Keywords: parking occupancy sensor, pricing parking

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1. Introduction

Parking occupancy sensors (sensors henceforth) are small devices built into the surface of automobile parking bays. Their function is to detect the presence of a vehicle in that bay and to live-feed the information, wirelessly, to the motorists searching for parking. Such information makes the search for parking more directed and thereby decreases the search time, pollution, accidents, and traffic.

The sensor technology, however, is not free. For example, San Francisco's SFPark, a project that deployed over 5000 sensors throughout the city center, required about $27 million of funding over two years. Barcelona's Urbiotica cost was between $200 and $400 per sensor to install (Ross (2011)). It is therefore important to understand the factors driving the economic benefits of sensor installation, which is the primary focus of this paper.

The key questions answered by this paper is the interplay between two government policies: installation of parking sensors and pricing parking. While the latter problem has been extensively discussed in many studies, it has always focused either on homogenous supply of parking (as in Glazer and Niskanen (1992), Arnott and Inci (2010), Zakharenko (2016)) or on exogenous heterogeneity of parking supply (Anderson and de Palma (2004), Kobus et al. (2013), Inci and Lindsey (2015)). Because installation of sensors is a choice of a government, sensor technology creates an endogenous heterogeneity of parking supply. Understanding how such heterogeneity affects optimal pricing, and is optimally affected by pricing, especially in the presence of heterogenous demand for parking, is our goal. This paper shows that the mutual dependence of two government policies can be substantial.

To the best of my knowledge, this is the first study of economic aspects of parking occupancy sensors.

Methodologically, this paper develops a theoretical dynamic model of demand and supply for parking that allows heterogeneity of motorists with respect to their desired duration of parking and their value of a parking session. The model incorporates a costly search for a vacant parking space that may or may not be assisted by occupancy sensors. Motorists decide whether to search for parking endogenously; their decision is affected by the difficulty of search and by the monetary cost of parking. The resulting equilibrium can be managed by the government by means of price regulation, installation of additional parking sensors, and providing more parking space.

Three major questions are answered. Section 3 assumes an exogenous supply of sensored and non-sensored parking, and studies optimal pricing, as well as the optimal distribution of parkers between the two zones. Section 4 studies how the price for parking affects the economic incentives to install sensors – in absolute terms and relative to the incentives to expand parking space. Finally, section 5 investigates whether the incentives to install sensors are increasing or decreasing with the number of sensors already installed.

2. The model

The model of parking is elaborated from Zakharenko (2016). This is a model of a city area with geographically homogenous demand and supply of parking, set up in continuous time. For transparency of the arguments, all parameters of the current model are time-invariant.

On the supply side, there is a continuum of parking space of measure N, of which N_1 is equipped with sensors while the remaining $N_0 = N - N_1$ is not. We assume that sensored and non-sensored parking areas are geographically segregated from each other (e.g. located on neighboring streets, or on different sides of the same street) so the newly arriving travelers can choose the type of parking to be searched. At the same time, different types of parking are assumed to be sufficiently proximate to each other so the travelers get the same utility from being parked at either of them.

On the demand side, in any time interval $\mathrm{d}t$ there is a mass $B\mathrm{d}t$ of newly emerging travelers who consider whether to drive to the area in question and search for parking. Those who do decide to drive also have to choose the type of parking, sensored or non-sensored, to be searched. Once a vacant bay is found, each traveler parks for a fixed period τ and derives utility $v\tau$ from the parking session. The two parameters, τ and v, are realizations of random variables, drawn independently across travelers. Most of the time, we will assume an atomless distribution of τ and v with a p.d.f. $f(\tau, v)$. Once parked, travelers also have to pay a fee of p per unit of time. The cost of parking p is assigned by the government and is the key method of congestion regulation. Prices may differ across sensored and non-sensored zones.

If a traveler chooses to stay out, her outside opportunity is normalized to zero.

By a *flow* of travelers we will denote the mass of travelers that enter a parking zone, per unit of time.

In case of non-sensored parking, we assume that the inflowing travelers randomly and independently sample non-sensored parking bays, until a vacancy is found. Such assumption is common in parking economics literature, and is found, among others, in Zakharenko (2016), Geroliminis (2015), Anderson and de Palma (2004), and Arnott and Rowse (1999). Recently, Arnott and Williams (2017) criticize this approach, which they call a *binomial approximation*, for its neglect of spatial correlation of parking occupancy. For the purposes of this paper, however, the analytical tractability advantage of the binomial approximation outweighs its drawbacks. Assuming that the cost of a unit of search time is c, that a motorist samples r_0 parking bays per unit of time, and that non-sensored parking occupancy is q_0, the expected search cost is $\frac{c}{r_0}\frac{1}{1-q_0}$.

In case of sensored parking, the status of each parking bay is observed in real time. Nevertheless, search cost still exists because driving to a particular bay takes time, and so there is a non-trivial probability that a vacant bay becomes occupied before a searching traveler gets there. The search process may be very complex, with a traveler's chosen direction of search being dependent on real-time information about occupancy of a large number of parking bays around. Moreover, the chosen direction of search may have to be constantly updated as the occupancy status of nearby parking keeps changing. To keep the model analytically tractable, we assume that the search technology is the same binomial approximation as in the non-sensored parking, only with a higher rate r_1 of sampling of parking bays. Intuitively, a sensor allows a searching traveler to "sample" a bay without physically getting there. While such approximation may appear too crude, it indeed reflects the intuition that sensored parking allows travelers to find a vacant spot faster, given the same congestion levels.

For an atomless distribution of travelers over τ and v, the social welfare generation per unit of time is

$$W = B \int_0^\infty \tau \int_0^\infty v \sum_{i=0,1} I_i(\tau, v) f(\tau, v) \mathrm{d}v \mathrm{d}\tau - \sum_{i=0,1} \frac{c}{r_i} \frac{A_i}{1 - q_i}, \tag{1}$$

where $I_i(\tau, v) \in [0, 1]$ is the probability that a motorist with characteristics τ, v searches for parking of type i, with

$$I_1(\tau, v) + I_0(\tau, v) \le 1. \tag{2}$$

In (1), $A_i = B \int_0^\infty \int_0^\infty I_i(\tau, v) f(\tau, v) \mathrm{d}v \mathrm{d}\tau$ is the flow of new arrivals into parking zone i; $q_i = \frac{B}{N_i} \int_0^\infty \tau \int_0^\infty I_i(\tau, v) f(\tau, v) \mathrm{d}v \mathrm{d}\tau$ is the resulting occupancy in zone i.

We also make the following

Assumption 1. *The demand for parking is high enough so both types of parking are used in the social optimum,* $q_i > 0, i = 0, 1$.

3. The optimal regulation of parking

This section investigates the socially optimal regulation of parking demand, with an exogenously given amount of supply of both sensored and non-sensored parking. For the purposes of this section, we assume that the size of each zone, sensored and non-sensored, is exactly one half of the total supply: $N_i = \frac{1}{2} N$. For example, sensors can be installed on one side of the street. At the end of this section, we discuss the robustness of the results to relaxing this assumption.

The social planner maximizes (1), using $I_i(\tau, v)$ as controls, respecting the definitions of A_i and q_i, as well as the constraint (2). The optimal solution for a traveler with parameters τ, v is as follows: choose the maximum of three values, $\frac{\mathrm{d}W}{\mathrm{d}I_1(\tau,v)}$, $\frac{\mathrm{d}W}{\mathrm{d}I_0(\tau,v)}$, and 0, which correspond to parking in zone 1, zone 0, and opting to stay out, respectively. Observe that

$$\frac{\mathrm{d}W}{\mathrm{d}I_i(\tau, v)} = B f(\tau, v) \left[\tau v - \frac{c}{r_i} \frac{1}{1 - q_i} - \frac{c}{r_i} \frac{A_i}{N_i} \frac{\tau}{(1 - q_i)^2} \right]. \tag{3}$$

Denote by

$$C_i \equiv \frac{c}{r_i} \frac{1}{1 - q_i} \tag{4}$$

the expected cost of search of vacancy in zone i, and by

$$p_i \equiv \frac{c}{r_i} \frac{A_i}{N_i} \frac{1}{(1 - q_i)^2} \tag{5}$$

the externality of a parked vehicle in zone i on the searching vehicles, per unit of time. We will refer to p_i as the "price" of parking, as setting the price equal to p_i would internalize all externalities of a parked vehicle. Then, the optimal choice between zone i and the outside opportunity for a traveler with parameters τ, v is driven by the comparison of the expected net value of parking $v\tau - C_i - p_i\tau$ to zero (whichever is larger), while the choice between zones 1 and 0 is driven by the comparison of expected parking costs $C_1 + p_1\tau$ and $C_0 + p_0\tau$, whichever is smaller.

Observe that the total number of vehicles parked in zone i, $q_i N_i$, is also equal to $A_i \bar{\tau}_i$, where $\bar{\tau}_i$ is the mean duration of parking in zone i. This equality allows us to rewrite the socially optimal price p_i as follows:

$$p_i = \frac{c}{r_i} \frac{1}{\bar{\tau}_i} \frac{q_i}{(1 - q_i)^2}. \tag{6}$$

3.1. Price discrimination for non-sensored parking

Even without the sensor technology, the benefits of price discrimination of parking have not been properly studied. The predecessor of the current model, Zakharenko (2016), has only considered a uniform price so all parking sites are ex-ante equally attractive to those who search for parking. What if all parking is segregated into two zones, labeled 1 and 0, with the same search technology $r_1 = r_0$, such that both zones are geographically equally attractive to all parkers? Would it make sense to price the two zones differently? Some studies, e.g. Anderson and de Palma (2004), Arnott and Rowse (2009) have considered geographic segregation of parking, but no study to my knowledge has discussed whether it is beneficial to segregate parking *at the same location* into short-term and long-term. We now analyze this question, as a necessary prerequisite to understanding price discrimination between sensored and non-sensored parking.

Suppose first that the socially optimal price (6) is the same in both zones, $p_1 = p_0$. Then, Assumption 1 requires that

$$\frac{c}{r_1}\frac{1}{1-q_1} = C_1 = C_0 = \frac{c}{r_0}\frac{1}{1-q_0}, \tag{7}$$

as well (otherwise one of the zones would be cheaper for all parkers and no one would demand the other). Recalling the assumption that $r_1 = r_0$, we conclude that in equilibrium the expected occupancy must be the same, too: $q_1 = q_0$. Then, all parkers are completely indifferent between the two zones, and can be directed to any of them without any effect on welfare, as long as the equality of occupancy is preserved. Suppose all parkers whose duration of parking τ is under some τ^* are directed to zone 1, while the rest of parkers go to zone 0. The cutoff τ^* is determined by equality of occupancy in the two zones. Obviously, $\tau_1 \leq \tau^* \leq \tau_0$, with at least one inequality being strict when there is non-zero variance of τ.

The optimal price in zone i is determined by (6). Because we have that $q_1 = q_0$, the equality of optimal prices $p_1 = p_0$ can be held only when parking duration is homogenous so that $\bar{\tau}_1 = \bar{\tau}_0$. In case of heterogeneity of τ, we have that $\bar{\tau}_1 < \bar{\tau}_0$, hence the initial assumption $p_1 = p_0$ is not socially optimal.

This finding suggests that, even without parking sensors, it is beneficial to make fraction of parking capacity more expensive. Then, short-term parkers benefit from shorter search of vacancy in the expensive zone, while the long-term parkers benefit from lower cost of the cheap zone. To the best of my knowledge, this result is novel by itself: no prior study has discussed the benefits of price discrimination for parking that is geographically homogenous.

We use this result as a starting point for the analysis that follows.

3.2. Partly sensored parking

Throughout the rest of the paper, we assume $r_1 > r_0$ due to installed sensors in zone 1. We now introduce several results that characterize the social optimum.

Lemma 1. *If $C_i < C_j, i, j \in \{0, 1\}$, then (i) $p_i > p_j$, and (ii) there is a cutoff parking duration τ^* so that those with $\tau < \tau^*$ park, if anywhere, in zone i, while those with $\tau > \tau^*$ park, if anywhere, in zone j.*

Proof. If $p_i \leq p_j$, then the total cost of parking is strictly lower for all travelers in zone i, so no one parks in zone j, contradicting the Assumption 1. This proves (i). Given $p_i > p_j$, there is a unique $\tau^* = \frac{C_j - C_i}{p_i - p_j} > 0$ such that $\tau < \tau^*$ is associated with lower cost in zone i, while $\tau > \tau^*$ is associated with lower cost in zone j. ■

Lemma 2. *The socially optimal cost of search is strictly lower in the sensored zone, $C_1 < C_0$.*

Proof. We prove the result by assuming the opposite,

$$\frac{c}{r_1}\frac{1}{1-q_1} = C_1 \geq C_0 = \frac{c}{r_0}\frac{1}{1-q_0}, \tag{8}$$

and arguing to a contradiction. Observe that such inequality implies $q_1 > q_0$. Also, to meet Assumption 1, (8) implies $p_1 \leq p_0$, with equality iff (8) is held with equality. Recalling the expression (6), and dividing the inequality $p_1 \leq p_0$ by the inequality (8), we obtain

$$\frac{1}{\bar{\tau}_1}\frac{q_1}{1-q_1} \leq \frac{1}{\bar{\tau}_0}\frac{q_0}{1-q_0}, \tag{9}$$

with equality iff (8) is held with equality. Together with $q_1 > q_0$, the inequality (9) implies

$$\bar{\tau}_1 > \bar{\tau}_0. \tag{10}$$

In case of homogenous τ, (10) is impossible, and so is (8).

Consider now heterogenous parking durations τ. Consider the following relocation between zones 1 and 0: all of zone-0 parkers are relocated to zone 1, while the flow $A_1 - \epsilon = \frac{q_1 N_1}{\bar{\tau}_1} - \epsilon$, with average parking duration $\bar{\tau}_1$, are relocated from zone 1 to zone 0. Thus, zone 1 mixes (i) a small flow ϵ of parkers with average duration $\bar{\tau}_1$ that used the same zone before with (ii) the flow $A_0 = \frac{q_0 N_0}{\bar{\tau}_0}$ of all parkers previously using zone 0.

The new average parking duration in zone 1 is $\bar{\tau}_1' = \frac{A_0 \bar{\tau}_0 + \epsilon \bar{\tau}_1}{A_0 + \epsilon} = \frac{q_0 N_0 + \epsilon \bar{\tau}_1}{\frac{q_0 N_0}{\bar{\tau}_0} + \epsilon}$, while the new occupancy is $q_1' = \frac{(A_0+\epsilon)\bar{\tau}_1'}{N_1} = q_0 + \frac{\bar{\tau}_1 \epsilon}{N_1}$. The latter equality makes use of our earlier assumption that the two zones have equal size, $N_1 = N_0$.

In zone 0, the new mean parking duration is $\bar{\tau}_0' = \bar{\tau}_1$ and the new occupancy is $q_0' = \frac{(A_1 - \epsilon)\bar{\tau}_1}{N_0} = q_1 - \frac{\bar{\tau}_1 \epsilon}{N_1}$.

We now analyze how the relocation affects the social welfare. Because such relocation does not affect the number and duration of parking sessions, the only possible effect is through the social cost of search for parking. The initial cost, before relocation, is

$$TC_0 = \sum_{i=0,1} \frac{c}{r_i} \frac{A_i}{1-q_i} = N_1 \frac{c}{r_1} \frac{1}{\bar{\tau}_1} \frac{q_1}{1-q_1} + N_0 \frac{c}{r_0} \frac{1}{\bar{\tau}_0} \frac{q_0}{1-q_0}. \tag{11}$$

The new social search cost is

$$TC(\epsilon) = \sum_{i=0,1} N_i \frac{c}{r_i} \frac{1}{\bar{\tau}_i'} \frac{q_i'}{1-q_i'} = \frac{c}{r_1} \frac{\frac{q_0 N_0}{\bar{\tau}_0} + \epsilon}{1 - q_0 - \frac{\bar{\tau}_1 \epsilon}{N_1}} + N_0 \frac{c}{r_0} \frac{1}{\bar{\tau}_1} \frac{q_1 - \frac{\tau_1 \epsilon}{N_1}}{1 - q_1 + \frac{\tau_1 \epsilon}{N_1}}.$$

First, we compare $TC(0)$ to TC_0. We have that (recalling $N_1 = N_0$)

$$TC(0) - TC_0 = N_1 c \left(\frac{1}{r_0} - \frac{1}{r_1} \right) \left[\frac{1}{\bar{\tau}_1} \frac{q_1}{1-q_1} - \frac{1}{\bar{\tau}_0} \frac{q_0}{1-q_0} \right]. \tag{12}$$

Observe that the term in round brackets is strictly positive, while the sign of the term in the square brackets is determined by (9). If $C_1 > C_0$, (9) is a strict inequality, (12) is strictly negative, thus the relocation of parking flows has reduced social search costs, contradicting the initial assumption of optimality.

If $C_1 = C_0$, then (9) is held with equality, thus $TC(0) - TC_0 = 0$, meaning that the relocation of flows with $\epsilon = 0$ does not change welfare. Consider the derivative of $TC(\cdot)$ at $\epsilon = 0$:

$$TC'(0) = \frac{c}{r_1}\frac{1}{1-q_0}\left[1 + \frac{\bar{\tau}_1}{\bar{\tau}_0}\frac{q_0}{1-q_0}\right] - \frac{c}{r_0}\frac{1}{(1-q_1)^2}. \tag{13}$$

Using the fact that (9) is equality, we can rewrite (13) as

$$TC'(0) = \frac{1}{1-q_1}\left(\frac{c}{r_1}\frac{1}{1-q_0} - \frac{c}{r_0}\frac{1}{1-q_1}\right),$$

which is strictly negative due to $r_1 > r_0$ and $q_1 > q_0$. Therefore, for small positive ϵ we have $TC(\epsilon) < TC(0) = TC_0$, thus we have achieved an improvement over the initial allocation of parking flows, compromising its optimality. ■

We now arrive at the main result of this section.

Proposition 1. *The sensored zone has a higher optimal price for parking and is used by shorter-term parkers.*

The proof follows trivially from Lemmas 1 and 2. Therefore, there exists a threshold parking duration τ^* such that travelers with $\tau < \tau^*$ will park, if anywhere, in the sensored zone, while those with $\tau > \tau^*$ will park, if anywhere, in the non-sensored zone. In math, $I_1(\tau, v) = 0$ for $\tau > \tau^*$ and $I_0(\tau, v) = 0$ for $\tau < \tau^*$.

The results of this section relied on the assumption that parking is divided into two zones, sensored and non-sensored, of equal size. The results can be easily extended to n zones of equal size, of which k are sensored. In this case, one can show that it is optimal to segregate parkers by duration of parking into n groups, and to allocate k shortest-duration groups into the sensored zones. We omit formal analysis because segregation of parking at the same geographic location into many zones does not appear to be viable.

At the same time, the results of this section may not extend to the case when there are two zones of *unequal* size. This is because inequality of zone size creates an additional incentive to match size heterogeneity of different groups of parkers to the size heterogeneity of the parking zones; this new incentive may oppose and prevail the incentive to match short-term parkers to sensored zones. Consider the following example. Suppose the sensored zone is much smaller than the non-sensored zone, $N_1 \ll N_0$. At the same time, suppose the flow of long-term parkers is very small compared to the flow of short-term parkers. Then, it may be optimal to direct the small (long-term) group to the small (sensored) area, by imposing a low price there, thereby violating Proposition 1.

4. The effect of price regulation on sensor installation incentives

Demand for parking is greatly affected by the price for parking. Many cities around the world offer parking that is too cheap, which leads to excess demand and shortage of available parking space. Low price also leads to cruising for parking, i.e. driving around the destination location and looking for vacant parking.[1] To address the problem, such cities often increase the supply of parking, either by investing public funds or by mandating private landowners to supply a certain amount of parking. Shoup (2005) provides an extensive account of social costs caused by excess incentives to build parking, that are in turn caused by suboptimal price.

This section investigates how the incentives to install occupancy sensors are affected by price regulation. To keep variation in such regulation unidimensional, we will assume that the government charges the same price p per unit of time for both sensored and non-sensored parking. Although Section 3.2 recommends price discrimination between the two types of

[1] There is a substantial literature that addresses the social costs of cruising for parking. For example, Van Ommeren et al. (2011) assess the magnitude of the problem empirically. Inci (2015) reviews the theory.

parking, its conclusions are only applicable when sensored and non-sensored zones have the same size. Such assumption is not suitable for this section, as we will consider marginal changes in parking supply in the presence of heterogenous demand.

Uniform pricing also makes calculations more simple, as it makes the cost of a parking session the same in both zones, so the newly arriving parkers search wherever the expected duration of search is shorter. Since such expected durations are the same for parkers of all types, in equilibrium they must also be equal to each other:

$$\frac{1}{r_1}\frac{1}{1-q_1} = \frac{1}{r_0}\frac{1}{1-q_0},$$

which allows to relate the occupancy in the two zones as follows: $q_1 = 1 - \frac{r_0}{r_1}(1-q_0)$. The occupancy q_0 is in turn determined by the equality of demand and supply of parking, as follows:

$$G(p, q_0) \equiv -B\int_0^{\infty} \tau \int_{v^*(p,q_0,\tau)}^{\infty} f(\tau, v)\mathrm{d}v\mathrm{d}\tau + \left(1 - \frac{r_0}{r_1}(1-q_0)\right) N_1 + q_0 N_0 \equiv 0, \qquad (14)$$

where

$$v^*(p, q_0, \tau) \equiv p + \frac{1}{\tau}\frac{c}{r_0}\frac{1}{1-q_0} \qquad (15)$$

is the value entry threshold of a traveler demanding a parking session of duration τ. We have that $\frac{\partial G}{\partial p} > 0$ and $\frac{\partial G}{\partial q_0} > 0$, hence, not surprisingly, $\frac{\partial q_0}{\partial p} = -\frac{\frac{\partial G}{\partial p}}{\frac{\partial G}{\partial q_0}} < 0$.

The generation of social welfare from the parking process, per unit of time, can be modified from (1) as follows:

$$W(p, q_0) \equiv B\int_0^{\infty} \tau \int_{v^*(p,q_0,\tau)}^{\infty} v f(\tau, v)\mathrm{d}v\mathrm{d}\tau - B\int_0^{\infty}\int_{v^*(p,q_0,\tau)}^{\infty} f(\tau, v)\mathrm{d}v\mathrm{d}\tau\frac{c}{r_0}\frac{1}{1-q_0}. \qquad (16)$$

4.1. The relative return to sensored parking

First, we investigate the relative social return to sensored parking, that is, the absolute social return to sensored parking $\frac{\mathrm{d}W}{\mathrm{d}N_1}$, relative to that of non-sensored parking $\frac{\mathrm{d}W}{\mathrm{d}N_0}$. We are specifically interested in how such relative return is affected by the price for parking p. The merit of such analysis is as follows. Suppose the government is determined to spend a certain amount of funds on improving parking supply and is choosing between (i) the low-tech improvement, i.e. increasing non-sensored parking capacity N_0, and (ii) the high-tech improvement, i.e. installing additional occupancy sensors, thereby increasing N_1 and decreasing N_0 by the same amount. Such government is more likely to choose the high-tech investment when the relative return to sensored parking is higher, so it is important to understand how the price regulation affects such return.

The absolute social return to expanding parking of type $i \in \{0, 1\}$ can be calculated as follows:

$$\frac{\mathrm{d}W}{\mathrm{d}N_i} = \frac{\partial W}{\partial q_0}\frac{\partial q_0}{\partial N_i} = -\frac{\partial W}{\partial q_0}\frac{\frac{\partial G}{\partial N_i}}{\frac{\partial G}{\partial q_0}}. \tag{17}$$

The relative social return can then be calculated as follows:

$$\frac{\frac{\mathrm{d}W}{\mathrm{d}N_1}}{\frac{\mathrm{d}W}{\mathrm{d}N_0}} = \frac{\frac{\partial G}{\partial N_1}}{\frac{\partial G}{\partial N_0}} = \frac{1 - \frac{r_0}{r_1}(1-q_0)}{q_0} = \frac{1}{q_0}\frac{r_1 - r_0}{r_1} + \frac{r_0}{r_1}. \tag{18}$$

Because $r_1 > r_0$ and the occupancy q_0 is negatively affected by the price p, we conclude that a higher price increases the relative return to sensored parking. A government that chooses between more parking space and more sensors is more likely to choose the latter when the price for parking is higher.

4.2. The absolute return to sensored parking

Understanding the relative return to sensored parking is useful but not sufficient for optimal sensor installation decisions. A higher price implies a lower usage of parking facilities and thereby might decrease the returns to any upgrading of those facilities. In particular, the return to building additional parking space, N_0, indeed decreases with a higher price for parking.

In case of occupancy sensors, however, there exists a positive counter-effect that negates all or part of the negative effect. Specifically, the gap in occupancy levels, $q_1 - q_0 = \frac{r_1 - r_0}{r_1}(1 - q_0)$, decreases with occupancy level q_0 and thereby increases with price p. Thus, a higher price may produce a greater additional value of an occupancy sensor.

The goal of this section is to understand the absolute marginal returns to sensor installation, or the MRS (marginal return to a sensor) for short. We limit the analysis to a simplified version of a model, to keep it analytically tractable and insightful. Specifically, we will impose a restriction on the distribution of travelers by assuming that the density $f(\tau, v)$ is constant with respect to v within the range from zero to some upper bound $\bar{v}(\tau)$, for any given τ. We redenote such density by $f(\tau)$. In words, travelers who demand parking of duration τ are uniformly distributed with respect to their value v of a unit of parking time. Denote by $\theta = \frac{N_1}{N}$ the share of sensored parking. Given the fact that the welfare (16) does not depend on θ directly, only via changes in occupancy, the MRS can be calculated as

$$\frac{\mathrm{d}W}{\mathrm{d}\theta} = \frac{\partial W}{\partial q_0}\frac{\mathrm{d}q_0}{\mathrm{d}\theta} = -\frac{\partial W}{\partial q_0}\frac{\frac{\partial G}{\partial \theta}}{\frac{\partial G}{\partial q_0}}. \tag{19}$$

We now proceed to calculate each element of (19) separately.

$$\begin{aligned}
\frac{\partial W}{\partial q_0}(p, q_0) &= -B\int_0^\infty v^*(p, q_0, \tau) f(\tau)\mathrm{d}\tau \frac{c}{r_0}\frac{1}{(1-q_0)^2} + B\int_0^\infty \frac{1}{\tau}\frac{c}{r_0}\frac{1}{1-q_0} f(\tau)\mathrm{d}\tau \frac{c}{r_0}\frac{1}{(1-q_0)^2} \\
&- B\int_0^\infty \int_{v^*(p,q_0,\tau)}^{\bar{v}(\tau)} f(\tau)\mathrm{d}v\mathrm{d}\tau \frac{c}{r_0}\frac{1}{(1-q_0)^2} \\
&= -B\frac{c}{r_0}\frac{1}{(1-q_0)^2}\int_0^\infty \left[p + \bar{v}(\tau) - v^*(p, q_0, \tau)\right] f(\tau)\mathrm{d}\tau \\
&= -B\frac{c}{r_0}\frac{1}{(1-q_0)^2}\int_0^\infty \left[\bar{v}(\tau) - \frac{1}{\tau}\frac{c}{r_0}\frac{1}{1-q_0}\right] f(\tau)\mathrm{d}\tau;
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial G}{\partial \theta} &= \frac{r_1 - r_0}{r_1}(1-q_0)N, \\
\frac{\partial G}{\partial q_0} &= B\frac{c}{r_0}\frac{1}{(1-q_0)^2} + \left[\frac{r_0}{r_1}\theta + 1 - \theta\right]N.
\end{aligned}
$$

Denote $z = \frac{1}{1-q_0}$; it increases with q_0 from unity, which corresponds to $q_0 = 0$ and a prohibitively high price, to some z_m which corresponds to free parking. The MRS, (19), is then

$$
\frac{\mathrm{d}W}{\mathrm{d}\theta} = B\frac{c}{r_0}\frac{r_1 - r_0}{r_1}N\frac{z\left(\int_0^\infty \bar{v}(\tau)f(\tau)\mathrm{d}\tau - \frac{c}{r_0}z\int_0^\infty \frac{1}{\tau}f(\tau)\mathrm{d}\tau\right)}{B\frac{c}{r_0}z^2 + \left[\frac{r_0}{r_1}\theta + 1 - \theta\right]N}. \tag{20}
$$

Note that (20) does not directly depend on the price for parking, only indirectly via z which depends negatively on p. Thus, the effect of p on $\frac{\mathrm{d}W}{\mathrm{d}\theta}$ can be calculated as $\frac{\mathrm{d}^2W}{\mathrm{d}\theta\mathrm{d}p} = \frac{\mathrm{d}^2W}{\mathrm{d}\theta\mathrm{d}z}\frac{\mathrm{d}z}{\mathrm{d}p}$. Denote the elements of (20) as follows: $E\bar{v}(\tau) = \int_0^\infty \bar{v}(\tau)f(\tau)\mathrm{d}\tau$, $E\frac{1}{\tau} = \int_0^\infty \frac{1}{\tau}f(\tau)\mathrm{d}\tau$, $k_d \equiv \left[\frac{r_0}{r_1}\theta + 1 - \theta\right]N$. Then, we can show that $\frac{\mathrm{d}^2W}{\mathrm{d}\theta\mathrm{d}z}$ has the same sign as

$$
E\bar{v}(\tau)k_d - 2\frac{c}{r_0}E\frac{1}{\tau}k_d z - E\bar{v}(\tau)B\frac{c}{r_0}z^2, \tag{21}
$$

which is strictly decreasing with z. We can distinguish three cases.

- At $z = 1$ (minimal value), (21) is non-positive: $E\bar{v}(\tau)k_d - 2\frac{c}{r_0}E\frac{1}{\tau}k_d - E\bar{v}(\tau)B\frac{c}{r_0} \leq 0$. (21) is then strictly negative at every $z > 1$. Such inequality can take place when the expected maximum value of parking $E\bar{v}(\tau)$ is small relative to the search cost c, and/or the parking capacity N is small relative to the potential inflow of parkers B. In such scenario, $\frac{\mathrm{d}^2W}{\mathrm{d}\theta\mathrm{d}z} < 0$ for every $z > 1$, hence the MRS monotonically increases with the price, $\frac{\mathrm{d}^2W}{\mathrm{d}\theta\mathrm{d}p} > 0$.

- At $z = 1$, (21) is positive, but at $z = z_m$ it is negative. Such scenario indicates that $E\bar{v}(\tau)$ is high relative to the search cost c, so one should expect high occupancy when parking is free, thus z_m is high. In this scenario, the MRS increases as p rises from zero to some intermediate value, and decreases thereafter.

- (21) is non-negative at $z = z_m$. It is then strictly positive at every $z < z_m$. This scenario is realized when parking supply N is large relative to the demand B, so the free-parking occupancy is low. In this case, the MRS monotonically decreases with the price, much like the marginal return to an additional parking space $\frac{\mathrm{d}W}{\mathrm{d}N}$.

5. Is marginal return to sensor increasing or decreasing?

How does the MRS depend on the number of sensors already installed? If the MRS is increasing or constant, corner solutions should be expected, i.e. parking facilities should be either fully sensored or fully non-sensored. With decreasing MRS, interior solutions (i.e. parking is partly sensored) are possible. This section focuses on this question.

The analysis below shows that the answer to the question is sensitive to the pricing method. We will consider two methods, uniform pricing and price discrimination, which produce the opposite results.

In this section, homogenous parking duration τ will be assumed, as I believe that the benefits of any additional insights of a more general model do not outweigh the costs of its additional complexity. Given the simplifying assumption, heterogeneity of parkers becomes unidimensional, over the value of parking v, with a p.d.f. $f(v)$.

5.1. Uniform pricing

Suppose the government sets the uniform welfare-maximizing price for parking. Because such price affects both equilibrium (14) and welfare (16) only via the scalar entry cutoff (15), we can treat this cutoff (rather than the price itself) as the control variable used by the government to maximize welfare. Given this assumption, we can rewrite the constraint (14) as follows:

$$G \equiv -B\tau \int_{v^*}^{\infty} f(v)\mathrm{d}v + \left(1 - \frac{r_0}{r_1}(1 - q_0)\right) \theta N + q_0(1 - \theta)N \equiv 0, \tag{22}$$

from which we can derive the following: $\frac{1}{1-q_0} = \frac{N\left(\frac{r_0}{r_1}\theta+1-\theta\right)}{N-B\tau\int_{v^*}^{\infty} f(v)\mathrm{d}v}$. By plugging the latter into (16), we can express welfare as follows:

$$W(v^*,\theta) = B\tau\int_{v^*}^{\infty} vf(v)\mathrm{d}v - B\frac{c}{r_0}\frac{N\left(\frac{r_0}{r_1}\theta+1-\theta\right)\int_{v^*}^{\infty} f(v)\mathrm{d}v}{N-B\tau\int_{v^*}^{\infty} f(v)\mathrm{d}v}. \tag{23}$$

The government selects v^* to maximize welfare. Note that v^* cannot optimally equal the upper bound of v as it implies zero entry. Assuming a sufficiently high demand for parking $B\tau$, v^* cannot equal the lower bound of v as it would result in a violation of the equilibrium condition (22) for every $q_0 \leq 1$. Thus, we will seek for an interior solution of v^*, so that the first-order condition of optimality is

$$\frac{\partial W}{\partial v^*} = 0. \tag{24}$$

We can show that such solution is unique so it implicitly defines v^* as a function of θ. We can also show that (24) being true also implies $\frac{\partial^2 W}{\partial v^{*2}} < 0$, so the second-order condition of optimality is also met.

We now investigate the convexity of welfare with respect to θ. Because v^* is chosen to maximize welfare, by the envelope theorem, the first derivative is

$$\frac{\mathrm{d}W}{\mathrm{d}\theta} = \frac{\partial W}{\partial\theta} = B\frac{c}{r_0}\frac{N\frac{r_1-r_0}{r_1}\int_{v^*}^{\infty} f(v)\mathrm{d}v}{N-B\tau\int_{v^*}^{\infty} f(v)\mathrm{d}v}. \tag{25}$$

Note that (25) is affected by θ only via the entry cutoff v^*. The dependence of (25) on v^* is negative while v^* is affected by θ also negatively (more sensors allow more usage of parking, thus drops the entry cutoff). Therefore, we can conclude that the MRS is increasing, $\frac{\mathrm{d}^2 W}{\mathrm{d}\theta^2} > 0$.

5.2. Price discrimination

Suppose now that the government sets the optimal price, given by (6), in each of the two zones separately. Given homogeneity of τ, the optimal entry value cutoff is given by

$$v^{**} = p_i + \frac{1}{\tau}\frac{c}{r_i}\frac{1}{1-q_i} = \frac{1}{\tau}\frac{c}{r_i}\frac{1}{(1-q_i)^2}, i \in \{0, 1\}. \tag{26}$$

The double asterisk for v^{**} emphasizes that this entry threshold is welfare-maximizing. From (26), we can derive the following expression for the occupancy q_i as a function of v^{**}:

$$q_i(v^{**}) = 1 - \left(\frac{c}{\tau r_i v^{**}}\right)^{\frac{1}{2}}. \tag{27}$$

Then, the equilibrium in the market for parking space can be defined as follows (cf.(14)):

$$G(v^{**}, \theta) = -B\tau \int_{v^{**}}^{\infty} f(v)\mathrm{d}v + q_1(v^{**})\theta N + q_0(v^{**})(1-\theta)N. \tag{28}$$

The social welfare generation per unit of time is (cf.(16))

$$W = B\tau \int_{v^{**}}^{\infty} vf(v)\mathrm{d}v - \sum_{i=0,1} A_i \frac{c}{r_i}\frac{1}{1-q_i}, \tag{29}$$

where A_i is the rate of arrival into zone i. It satisfies the following relationship: $A_i = \frac{1}{\tau}q_i(v^{**})N_i$ which, together with (27), allows us to rewrite (29) as follows:

$$W(v^{**}, \theta) = B\tau \int_{v^{**}}^{\infty} vf(v)\mathrm{d}v - \left[\left(\left(\frac{cv^{**}}{\tau r_1}\right)^{\frac{1}{2}} - \frac{c}{\tau r_1}\right)\theta + \left(\left(\frac{cv^{**}}{\tau r_0}\right)^{\frac{1}{2}} - \frac{c}{\tau r_0}\right)(1-\theta)\right] N. \tag{30}$$

The MRS is then

$$\frac{\mathrm{d}W}{\mathrm{d}\theta} = \frac{\partial W}{\partial \theta} + \frac{\partial W}{\partial v^{**}}\frac{\mathrm{d}v^{**}}{\mathrm{d}\theta} = \frac{\partial W}{\partial \theta} - \frac{\partial W}{\partial v^{**}}\frac{\frac{\partial G}{\partial \theta}}{\frac{\partial G}{\partial v^{**}}} \tag{31}$$

Observe that $-\frac{\frac{\partial W}{\partial v^{**}}}{\frac{\partial G}{\partial v^{**}}} = v^{**}$, which enables us to calculate (31) as follows:

$$\frac{\mathrm{d}W}{\mathrm{d}\theta} = 2\left(\frac{cv^{**}}{\tau}\right)^{\frac{1}{2}}\left(\frac{1}{r_0^{\frac{1}{2}}} - \frac{1}{r_1^{\frac{1}{2}}}\right)N + \frac{c}{\tau}\left(\frac{1}{r_1} - \frac{1}{r_0}\right)N. \tag{32}$$

A rise in the share of sensored parking θ drops the entry cutoff v^{**} which reduces the MRS given by (32). Thus, when price discrimination is used, we have that $\frac{\mathrm{d}^2W}{\mathrm{d}\theta^2} < 0$, in contrast with uniform pricing. However, the magnitude of $\frac{\mathrm{d}^2W}{\mathrm{d}\theta^2}$ is most likely very small, so the MRS does not vary too much, making it unlikely that the government chooses an interior solution for the share of sensored parking θ.

6. Conclusion

This paper is the perhaps the first attempt to look into the economic merits of parking occupancy sensors. We find that such merits are strongly affected by how parking is priced. In particular, this paper finds that excessively cheap parking dampens the incentives to install parking sensors, both in absolute terms and relative to parking-space-expansion incentives. This paper also finds that the reverse causality exists, too: pricing may optimally change when parking sensors are installed.

The theoretical analysis of this paper is based on a rather simplistic “binomial” approximation of the search-for-parking process. In a simulation study of non-sensored parking, Arnott and Williams (2017) highlight the fact that such approximation, by ignoring spatial correlation of parking occupancy, makes imperfect predictions about the aggregate statistics of the search process. In case of sensored parking, such approximation may depart even further from the real world, as the information fed by the sensors allows drivers to search strategically. Further simulation or empirical research is needed to understand how sensors affect the search process.

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