Abstract

The effects of autonomous vehicles (AVs) on urban forms are modeled and analyzed. Vehicles are used for commute between peripheral home and central work, and require space for day parking. An advantage of AVs is that they can return home for day parking, relieving central land for other uses. Lowered AV costs increase the mass of their users at the expense of traditional commuters and non-commuters. Worker welfare increases, land rents decrease, cities become smaller, but at least some groups of commuters have to travel longer distances. Potential modifications of the model are discussed, in which AVs cause expansion rather than contraction of cities.

Keywords: Self-driving cars, autonomous vehicles, commute, parking, urban forms

JEL codes: D61, O18, R41, R49

1. Introduction

The advent of self-driving cars, or autonomous vehicles (AVs henceforth) seems to be a matter of very near future. The rapidly growing literature, academic and non-academic, has discussed many virtues of AVs. They will dramatically reduce transport costs for the disabled. They will allow minors to travel without adults present. They will relieve passengers from the burden of driving, enhancing travel experience. They will travel more safely, choose the route more optimally, and will increase highway throughput. This paper focuses on yet another advantage of AVs: their ability to geographically separate themselves from their owners while not in use, in order to optimize parking costs. In doing so, AVs will relieve the
demand for centrally located parking, which will allow to reallocate all economic activity in
the city.

Cars indeed require a lot of space, and “traditional” cars that cannot drive themselves
typically require such space at every location their owner chooses to visit. As city centers
typically accommodate a large proportion of the city jobs, currently they should also ac-
commodate large amounts of parking capacity for those who travel to work by car. The
use of land for parking crowds out other land uses, eventually leading to reduced density of
economic activity. Shoup (2005, p. 130), and references therein, describe the extreme cases
of such crowding out. Downtown Buffalo, New York, allocates half of its land to parking.
Downtown Albuquerque, New Mexico, devotes to parking even more than half its land. And
in downtown Topeka, Kansas, the share of land dedicated to non-parking uses is so small
that, the author ironizes, there may be little reason to travel there and park.

When AVs appear on stage, cities may change dramatically. The new cars will be able to
travel back home for parking. Downtowns, with parking space removed, will see the increase
in density of economic activity, allowing to boost productivity. As the economic activity
will be drafted to downtowns from more peripheral locations, the latter will become more
available for residential use. As a result, cities will become more compact, more productive,
and will enjoy higher levels of welfare.

The objective of this paper is to formulate the above ideas rigorously within a model,
enabling formal analysis of the effects of AVs on urban land use. The model developed in
this paper gives answers to the following questions: when AVs emerge, where in the city
will their owners work (relative to other workers) and live (relative to other residents)? How
will other commute modes, such as travel by a traditional car or no travel at all, be affected
by such innovation? What are the effects on worker consumption, land rents, and commute
distances?

The existing literature that analyses the AVs introduction from the urban economics
perspective is scant. Fagnant and Kockelman (2015), in a survey of benefits and difficulties related to introduction of AVs, dedicate only one paragraph to land use issues. Zhang et al. (2015) simulate a city with random trip demand generation, to show a dramatic downward effect of shared AVs on parking demand. Their study however does not analyze how urban forms will be affected, by assuming exogenous location of travelers. Hayes (2011) points out that AVs will be able to park closer to each other, saving urban space. No other academic studies on the topic could be found.

Perhaps the most detailed analysis on the topic is an opinion article by Romem (2013). The author argues that (i) by reducing travel costs, AVs will make cities larger and (ii) by removing central parking, central economic activity will become more dense.

2. Model

2.1. Geography and Population

Consider a unidimensional city located on a line from zero to infinity. At point $a = 0$, there is a “port,” the only communication with the rest of the world. There are two goods in the economy, labeled as the “export” and the “import” good, that are exchanged one for another at the port. Throughout the paper, we refer to locations closer to the port (with smaller $a$) as more central, while the locations away from the port (with larger $a$) are peripheral.

The total labor force is equal to $L$.

2.2. Production

Production of one unit of the export good requires one unit of labor. I assume that the workers, each having one unit of labor, also serve as entrepreneurs who organize production by contributing their own labor and collecting the sales revenues.

The output of the export industry is not demanded domestically and must be delivered to the port. Transportation of the export good within the city is subject to iceberg transport.
cost: for one unit to arrive to the port, \( \frac{1}{1-\tau a} \) must be shipped from location \( a \), with \( \tau > 0 \). Thus, any production must be located within \( a \in [0, \frac{1}{\tau}) \). The price of the export good at the port is normalized to unity, hence the (perfectly competitive) price at location \( a \) is \( 1 - \tau a \).

The import good cannot be produced domestically and must be delivered from the port.

2.3. Consumption

The owners of labor and land are consumers who demand the import good. The import good is available at the port at the normalized price of unity, and can be delivered to any location in the city at no additional cost.\(^2\)

Land owners demand nothing else and locate themselves at the land they own, though not occupying any of it and making all of it available for rent.

The workers also require \( \gamma \) units of land for residence. The residential land is rented from the land owners. If the place of residence is different from the place of work, they have to commute, which incurs additional costs outlined below. Due to free mobility, consumption net of all costs must be the same for all workers, regardless of place of work and residence.

2.4. Commute technology

All workers own a vehicle which they may or may not use for commute between work and home. Vehicles require parking, creating an additional demand for land. We assume that residential (night) parking is bundled with everyone’s residence (e.g. by law) and is included into the cost of renting a residence.

There are several possible commute modes that we now outline.

2.4.1. No commute

One possible mode of commute is no commute at all, i.e. the workers living and working at the same place. In this case, workers do not use their vehicles during work and no

\(^1\)Adding the transport cost for the import good would further increase concentration of economic activity near the port. It would also reduce the consumer incentive to reside in the periphery.
additional costs are involved. We label the workers using this mode of transportation as non-commuters; their density at location \( a \) by \( x_N(a) \).

2.4.2. Traditional vehicles

Traditional vehicles allow to separate the place of work from the place of residence. There are two types of associated costs:

(i) Variable cost \( \delta_T \) per unit of distance of roundtrip commute. This includes both the vehicle operation cost and the opportunity cost of occupants’ time in transit.

(ii) Parking cost: each car requires \( \pi \) units of land at the place of work for day parking, such that \( \pi < \gamma \). A vacant residential parking spot may be used for parking, too.

The workers using this mode of transportation are labeled as traditional commuters. The density of such commuters working at \( a \) is denoted \( x_B(a) \); the density of such commuters residing at \( a \) is \( x_{RT}(a) \).

2.4.3. Autonomous vehicles

While there may be many unique characteristics of autonomous vehicles, we focus on one: their ability to park themselves away from their owners. To simplify analysis, we assume that (i) at night, an AV must be parked near home, just like a traditional vehicle, and (ii) the cost of travelling empty is low enough so an AV can travel home for day parking, i.e. locations of day and night parking are always the same. For this reason, there are no additional parking costs involved.

The associated costs of AV use are as follows:

(i) There are two variable costs per unit of travel distance. When the car carries passengers, the cost is \( \delta_{S0} \), which includes the opportunity cost of occupants’ time in transit. Because the occupants no longer have to do the driving job, it is reasonable to assume \( \delta_{S0} < \delta_T \). When the car travels empty, the cost is \( \delta_{S1} \) which is even lower than
\( \delta_{S0} \), because no one’s time is being spent in transit. Since, by assumption, the car returns to residential area for day parking and therefore travels empty the same distance as it travels with passengers, the total roundtrip cost per unit of such distance is 
\[ \delta_{S} = \delta_{S0} + \delta_{S1}. \]

(ii) Because AVs are likely to be more expensive than traditional vehicles, we assume there is an additional fixed cost \( \rho \) of an AV ownership. A possible additional utility of AVs from non-commute uses has a downward impact on \( \rho \).

We label the workers using AVs as *new commuters*. As they do not require any land for work, they are likely to concentrate their production at one location which we denote \( a_S \). The total mass of new commuters working at \( a_S \) is labeled \( L_S \). The residential density of new commuters at location \( a \) is labeled \( x_{RS}(a) \). It is also the density of both day and night parking for new commuters.

Note that the relation of \( \delta_{S} \) to \( \delta_{T} \) is ambiguous: on the one hand, new commuters can travel with less effort; on the other, vehicle-kilometers traveled are doubled as the car returns home for parking. We assume that the latter feature outweighs so that

\[ \delta_{S} > \delta_{T}. \]  
(1)

In addition, we assume that

\[ \tau > \delta_{S}, \]  
(2)

meaning that the productive gain from moving a new commuter workplace closer to the port exceeds the additional commute cost.

2.5. General considerations

The model satisfies the assumptions of the first Welfare theorem, therefore no Pareto-improvement should be possible in equilibrium. Since the utilities are defined over the
quantity of the import good and are thus transferable, any market equilibrium maximizes the aggregate consumption of the import good. Therefore, to find an equilibrium it is sufficient to solve the social planner’s problem of such consumption maximization. The shadow values of labor and land at each location are equal to wages and rents, respectively.

The next two sections characterize the solution to the problem.

3. Analysis

We now characterize several results that help to simplify the subsequent analysis.

**Proposition 1.** In equilibrium, a commuter worker’s place of residence $a_2$ is more peripheral than her place of work $a_1$: $a_1 < a_2$.

Intuitively, transportation costs of the export good make the workplace naturally gravitate towards the center, while no such gravity exists for the place of residence. The result holds for both traditional and new commuters.

**Proof.** First, we rule out $a_1 = a_2$: that would enable the commuter to turn into a non-commuter, dropping all commute costs without sacrificing any output. Next, we rule out $a_1 > a_2$: that would enable to move production to $a_2$, increasing output and eliminating commute costs. □

**Corollary 1.** The total commute distance (i.e. vehicle-kilometers traveled) is equal to the distance between the mean location of commuter residence and their mean location of work, multiplied by the mass of commuters, $\int_a a(x_{RT}(a) - x_B(a))da$ for traditional commuters and $\int_a ax_{RS}(a)da - L_S a S$ for new ones.

**Proposition 2.** The same location cannot be used for both place of work and place of residence of traditional commuters: $x_{HT}(a)x_{RT}(a) = 0$ at every $a$.

**Proof.** Suppose the contrary, that there exists a location $a_0$ at which “group 1” of traditional commuters works, while some other “group 2” of traditional commuters resides. Suppose
each of the two groups has mass $\Delta$. Group 1 resides and night-parks at some $a_1$, while group 2 works and day-parks at some $a_2$. By Proposition 1, we have that $a_2 < a_0 < a_1$.

Consider the following relocation: the location of residence of groups 1 and 2 are swapped. Since group 1 now works and resides at the same location $a_0$, it becomes a non-commuter group. The demand for parking at $a_0$ is nullified. The increase in variable commute cost by group 2 is compensated by an equal decrease in such cost by group 1. All other costs, as well as total production of export good by the two groups, are unchanged. Thus, aggregate welfare has increased, which rules out the equilibrium status of the initial allocation. □

Proposition 2 implies that a potential complementarity of day and night parking for traditional commuters is never realized, and therefore the total demand for land at any location $a$ by traditional commuters is equal to $\pi x_B(a) + \gamma x_{RT}(a)$.

**Proposition 3.** New commuters work at the port: $a_S = 0$.

This is a direct corollary of (2) and of the fact that production requires no land.

Denote by $x(a) = \{x_N(a), x_B(a), x_{RT}(a), x_{RS}(a)\}'$ the vector of location-specific controls. We can formulate the welfare maximization problem as a version of the optimal control problem: maximize aggregate value of production net of commute costs,

$$W(x(\cdot), L_S) = (1 - \rho)L_S$$
$$+ \int_{a \in [0, \infty)} (1 - \tau a)(x_N(a) + x_B(a)) - \delta_T a(x_{RT}(a) - x_B(a)) - \delta_S ax_{RS}(a) da,$$

subject to the following constraints:

- The *spatial constraint* for every location $a$, limiting the demand for land by its supply:

$$b'x(a) \leq 1,$$

If one group is larger than the other, we consider only a fraction of the larger group equal in mass to the smaller group.
where \( b = \{\gamma, \pi, \gamma, \gamma\} \). Denote by \( r(a) \geq 0 \) the Lagrange multiplier for this constraint; in equilibrium, it is equal to the land value at location \( a \).

- **Demographic constraint**: the density of workers (commuting or not) aggregated over space does not exceed the population size,

\[
R_L(x(\cdot), L) \equiv L_S + \int_{a \in [0, \infty)} \{1, 1, 0, 0\}x(a)da \leq L, \tag{5}
\]

We assume that the labor force \( L \) is small enough so the constraint is binding at the optimal point. Denote by \( \sigma \geq 0 \) the corresponding Lagrange multiplier; in equilibrium, it is equal to the worker consumption/utility.

- **Residential constraint for traditional commuters**: the residential capacity available to traditional commuters matches the work capacity,

\[
R_T(x(\cdot)) \equiv \int_{a \in [0, \infty)} \{0, 1, -1, 0\}x(a)da \leq 0. \tag{6}
\]

By \( \lambda_T \geq 0 \) we denote the Lagrange multiplier for the constraint; its economic meaning is the total cost of residence (cost of rent, plus the cost of commute to the residence) for a traditional commuter working at the port.

- **An analogous residential constraint for new commuters**:

\[
R_S(x(\cdot), L) \equiv L_S + \int_{a \in [0, \infty)} \{0, 0, 0, -1\}x(a)da \leq 0. \tag{7}
\]

The corresponding Lagrange multiplier is denoted \( \lambda_S \geq 0 \).

The above problem belongs to the class of optimal control problems because it has a continuous exogenous location variable \( a \). Although it does not have state variables, it does have isoperimetric constraints (see Chachuat (2007) for a definition) given by (5)–(7), as well
as inequality constraint (4) at every location. Since the objective function and all constraints are linear with respect to control variables $x(\cdot), L_S$, the problem satisfies the Mangasarian sufficient condition, and any solution of the problem delivers the global maximum.

The standard solution technique involves maximizing the Lagrangian

$$
\mathcal{L}(x(\cdot), L_S) \equiv W(x(\cdot), L_S) + \int_a r(a) (1 - b'x(a)) da + \sigma(L - R_L(x(\cdot), L_S)) - \lambda_T R_T(x(\cdot)) - \lambda_S R_S(x(\cdot), L_S)
$$

(8)

with respect to $x(a), L_S$.

The first-order condition for optimal $L_S$ reads

$$
1 - \rho - \sigma - \lambda_S \leq 0,
$$

(9)

with equality if $L_S > 0$.

We next analyze the first-order condition for each element of $x$ in greater detail.

**Non-commuter zone.** Denote by $A_N$ all locations where non-commuters are present, i.e. $x_N(a) > 0$. To ensure that $\frac{\partial H}{\partial x_N} = 0$ at such locations, the rent must satisfy $r(a) = r_N(a), \forall a \in A_N$, where

$$
r_N(a) = \frac{1 - \tau a - \sigma}{\gamma}.
$$

(10)

Outside of $A_N$, the condition $\frac{\partial H}{\partial x_N} \leq 0$ is equivalent to $r(a) \geq r_N(a), \forall a \notin A_N$.

**Traditional commuter work zone.** Denote by $A_B$ the locations where $x_B(a) > 0$. To meet the first-order condition of optimality, the rent satisfies $r(a) = r_B(a), \forall a \in A_B$, where

$$
r_B(a) = \frac{1 - \tau a + \delta_T a - \sigma - \lambda_T}{\pi}.
$$

(11)

Outside of $A_B$, we have that $r(a) \geq r_B(a), \forall a \notin A_B$. 

10
Traditional, New commuter residential zone. Denote by $A_{Rk}, k \in \{T, S\}$ the locations where $x_{Rk}(a) > 0$. Then, $r(a) = r_{Rk}(a), \forall a \in A_{Rk}$ and $r(a) \geq r_{Rk}(a), \forall a \notin A_{Rk}$, where

$$r_{Rk}(a) = \frac{-\delta_k a + \lambda_k}{\gamma}.$$  \hspace{1cm} (12)

Empty zone. Denote by $A_0$ the locations where no economic activity takes place: $x(a) = 0$. To meet the Kuhn-Tucker condition $r(a) (1 - b'x(a)) = 0$, the rent must be equal to zero: $r(a) = 0$.

Summarizing the above discussion, we have that

$$r(a) = \max\{r_N(a), r_B(a), r_{RT}(a), r_{RS}(a), 0\}.$$  \hspace{1cm} (13)

Since each argument of the maximum is linear, the rent $r(a)$ is a piecewise-linear, convex, and continuous function of space. In other words, the slope of $r(\cdot)$ must be increasing over space from some initial negative value to zero.

To refine analysis, we introduce the following additional assumptions:

Assumption 1.

$$\frac{\partial r_B(\cdot)}{\partial a} = -\frac{\tau - \delta_T}{\pi} < -\frac{\tau}{\gamma} = \frac{\partial r_N(\cdot)}{\partial a}.$$  \hspace{1cm} (14)

From earlier discussion, we also have that

$$\frac{\partial r_N(\cdot)}{\partial a} \lesssim \frac{\partial r_{RS}(\cdot)}{\partial a} \lesssim \frac{\partial r_{RT}(\cdot)}{\partial a} < 0.$$  \hspace{1cm} (cf. (2), cf. (1))

Assumption 2. All types of commute are practiced by a positive fraction of population in equilibrium. In particular, $L_S > 0$.

These assumptions together with properties of $r(a)$ imply that each of the zones $A_N, A_B, A_{RT}, A_{RS}, A_0$ is non-empty and connected. Also, any pair of these zones can have at most one
common point which is their joint boundary. We also have that the rent \( r(a) \) is strictly positive everywhere outside of \( A_0 \), meaning that the land is fully used at all such locations.

With these results, we can determine the density of economic activity at each of these zones:

\[
x_N(a) = \frac{1}{\gamma}, a \in \text{int}(A_N); x_B(a) = \frac{1}{\pi}, a \in \text{int}(A_B);
\]
\[
x_{RT}(a) = \frac{1}{\gamma}, a \in \text{int}(A_{RT}); x_{RS}(a) = \frac{1}{\gamma}, a \in \text{int}(A_{RS}). \tag{15}
\]

How are the zones located relative to each other? By Assumption 1 and by subsequent discussion we have that

\[
\sup A_B = \inf A_N < \sup A_N = \inf A_{RS} < \sup A_{RS} = \inf A_{RT} < \sup A_{RT} = \inf A_0.
\]

Define by \( a \equiv \{a_N, a_B, a_{RT}, a_{RS}\}' \) the suprema of \( A_N, A_B, A_{RT}, A_{RS} \), respectively. The location of these points, together with the three Lagrange multipliers \( \mu \equiv \{\sigma, \lambda_T, \lambda_S\}' \) and the mass of new commuters \( L_S \) can be found from the following system of equations:

- Optimization problem constraints (5, 6, 7), held with equality, rewritten as (cf 15)

\[
\begin{align*}
L_S + \frac{a_B}{\pi} + \frac{a_N - a_B}{\gamma} &= L \\
\frac{a_B}{\pi} - \frac{a_{RT} - a_{RS}}{\gamma} &= 0 \\
L_S - \frac{a_{RS} - a_N}{\gamma} &= 0.
\end{align*}
\]

- New commuter population optimality (9), held with equality.

- Continuity of land rent at borders of different zones: \( r_B(a_B) = r_N(a_B), r_N(a_N) = r_{RS}(a_N), r_{RS}(a_{RS}) = r_{RT}(a_{RS}), r_{RT}(a_{RT}) = 0. \)
This system can be rewritten in the matrix form

\[
\begin{pmatrix}
0_{3\times3} & e & B \\
0_{3\times4} & 0 \\
B' & 0_{4\times1} & D
\end{pmatrix}
\begin{pmatrix}
\mu \\
L_S \\
a
\end{pmatrix}
=
\begin{pmatrix}
c_\mu \\
1 - \rho \\
c_a
\end{pmatrix},
\]  

(16)

where \( e = \{1, 0, 1\}' \), \( B = \begin{pmatrix}
\frac{1}{\gamma} & \frac{1}{\pi} - \frac{1}{\gamma} & 0 & 0 \\
0 & \frac{1}{\pi} & -\frac{1}{\gamma} & \frac{1}{\gamma} \\
\frac{1}{\gamma} & 0 & 0 & -\frac{1}{\gamma}
\end{pmatrix} \), \( D = \text{diag}\{\frac{\tau - \delta_S}{\gamma}, \frac{\tau - \delta_T}{\pi} - \frac{\tau}{\gamma}, \frac{\delta_T}{\gamma}, \frac{\delta_S - \delta_T}{\gamma}\} \),

\( c_\mu = \{L, 0, 0\}' \), \( c_a = \{\frac{1}{\gamma}, \frac{1}{\pi} - \frac{1}{\gamma}, 0, 0\}' \).

Figure 1 illustrates the allocation of economic activity and the equilibrium rent as a function of space.
The solution to the above system of equations is given by

\[
L_S = \frac{1 - \rho + e' (BD^{-1}B')^{-1} [c_\mu - BD^{-1}c_a]}{e' (BD^{-1}B')^{-1} e}, \tag{17}
\]

\[
\mu = (BD^{-1}B')^{-1} \left[ BD^{-1}c_a - c_\mu + eL_s \right], \tag{18}
\]

\[
a = D^{-1} [c_a - B'\mu]. \tag{19}
\]

4. Increasing availability of self-driving cars

To fully understand how self-driving cars affect cities, we study how a marginal decrease in the fixed cost of the former, from \( \rho \) to \( \rho - \Delta \rho \), \( \Delta \rho > 0 \), affects all endogenous variables within the model. The changes in the unknowns can be calculated as follows:

\[
\Delta L_S = \frac{\Delta \rho}{e' (BD^{-1}B')^{-1} e}, \tag{20}
\]

\[
\Delta \mu = (BD^{-1}B')^{-1} e \Delta L_s, \tag{21}
\]

\[
\Delta a = -D^{-1} B' \Delta \mu. \tag{22}
\]

4.1. Mass of new commuters

Because \( D \) is a diagonal matrix with positive elements, we have that \( BD^{-1}B' \) is positive definite, which immediately implies that \( \Delta L_S > 0 \). In other words, cheaper self-driving cars increase the size of the community using them, which is hardly a surprise.

4.2. Consumption and costs

Regarding the elements of \( \Delta \mu \), we have the following results:

\[
\Delta \mu = \begin{pmatrix}
\Delta \sigma \\
\Delta \lambda_T \\
\Delta \lambda_s
\end{pmatrix} = \frac{\Delta L_s (\gamma - \pi)}{z \gamma^2 (\delta_s - \delta_T)((\gamma - \pi)\tau - \gamma \delta_T)} \begin{pmatrix}
\frac{\tau}{\delta_T} \\
-1 \\
\frac{(\gamma - \pi) \delta_s - \gamma \delta_T}{\pi \delta_T}
\end{pmatrix}, \tag{23}
\]
where \( z \equiv \det (BD^{-1}B') > 0 \). In (23), the multiplier outside of brackets is positive because of (II), Assumption \( \Pi \), and the fact that \( \pi < \gamma \).

Thus, \( \Delta \sigma \) is necessarily positive: workers consume more with greater AV availability. This result is very intuitive: the new technology of self-driving cars not only increases aggregate welfare, but also, by reducing the demand for parking land, reallocates this welfare from landowners to workers.

Next, we find that \( \Delta \lambda_T \), i.e. the total cost of residence ownership by traditional commuters decreases. Intuitively, residence zone \( A_{RT} \) of traditional commuters is a complement to their work zone \( A_B \), which, in turn, is a substitute to production by new commuters at the port. Cheaper AVs increase profitability of the latter and thereby decrease demand for the former.

Finally, the sign of \( \Delta \lambda_S \) is ambiguous: on the one hand, increasing community of new commuters increases the demand for land within \( A_{RS} \). On the other, more efficient land use by AVs decreases the cost of land throughout the city. The overall effect can be shown to be positive iff \( \frac{\gamma}{\pi} > \frac{\delta_S}{\delta_S - \delta_T} \).

Few additional results can be shown. First, \( \Delta \lambda_S - \Delta \lambda_T > 0 \): even if the rent within \( A_{RS} \) decreases, the drop is necessarily smaller than that in \( A_{RT} \). Second, \( \Delta \sigma + \Delta \lambda_T > 0 \), meaning cheaper parking within \( A_B \).

4.3. Zone boundaries

The change in zone boundaries is as follows:

\[
\Delta a = \left( \begin{array}{c}
\Delta a_N \\
\Delta a_B \\
\Delta a_{RT} \\
\Delta a_{RS}
\end{array} \right) = \frac{\Delta L_S(\gamma - \pi)}{z\delta_T\gamma^2(\delta_S - \delta_T)((\gamma - \pi)(t - \delta_T))} \begin{pmatrix}
-\gamma(\delta_S - \delta_T) - \pi(\tau - \delta_S) \\
\pi(\tau - \delta_S) \\
-1 \\
-1 \\
\frac{\gamma - \pi}{\pi}
\end{pmatrix}.
\]
From (24), the following conclusions can be made. First, the city is becoming smaller, as demonstrated by $\Delta a_{RT} < 0$. With more AVs in use, the central business district $A_B$ becomes smaller, allowing to move the residential district $A_N \cup A_{RS} \cup A_{RT}$ closer to the center. Second, the mass of commuters increases, as seen by the squeeze of the size of non-commuter zone $A_N = [a_B, a_N]$. Finally, we can see that the new commuter residence zone $A_{RS} = [a_N, a_{RS}]$ expands in both directions. The reallocation of zones and rent changes are described on Figure 2.

From (24), we can also draw inference about changes in commute distances. For traditional commuters, the mean travel distance is the difference between mean residence location, $\frac{1}{2} (a_{RT} + a_{RS})$, and the mean work location, $\frac{1}{2} a_B$. This difference is positive, meaning that, despite the fact that the city is getting smaller, traditional commuters on average drive longer distances. This is due to the fact that the outward boundary $a_B$ of the business district is pushed inwards, while the inward boundary of traditional commuter residence $a_{RS}$ is pushed outward.
For new commuters, the work location is at the port by Proposition 3, while the mean residence location is $\frac{1}{2}(a_{RS} + a_N)$. The change in the latter is ambiguous: it can be shown that if

$$\tau - \delta_S > \delta_S - \delta_T,$$

then the change in commute distance has the same sign as $\frac{2}{\pi} - 2 \left( \frac{\tau - \delta_S}{\tau - \delta_S - (\delta_S - \delta_T)} \right)$. If (25) is reversed, the commute distance unambiguously decreases. Intuitively, squeezing business district together with squeezing mass of non-commuters allows new commuter residences to move inward, but the rising mass of new commuters pushes their residence outward. The latter effect dominates if the squeeze of the non-commuter zone is slow enough (25), and if the ratio of (peripheral) residence requirement to (central) parking requirement $\frac{2}{\pi}$ is large enough.

The effect on travel distance aggregated across traditional and new commuters, clearly, depends on the share of each of these groups in population. When AVs just arrive and the share of new commuters is close to zero, the aggregate change in travel distance is equal to that of traditional commuters, i.e. is unambiguously positive.

5. Extensions

Unlike this paper, Romem (2013) argues that introduction of AVs will increase the city size. This is because, he argues, (i) people always want more space, and (ii) travel will be cheaper. The model developed in this paper predicts the opposite, that cities will shrink, because it assumes (i) fixed amount of space for each resident and (ii) AVs return home for parking, actually increasing the total cost of travel. We now discuss how these features can be relaxed, and whether the conclusions of Romem (2013) can be replicated.

If the cost of empty AV travel, $\delta_{S1}$, is high enough, new commuters may find it beneficial to day-park their vehicles somewhere between the location of work and the location of residence. Then, a “parking belt” just outside of the central business district may emerge. If all AVs
are day-parked at such parking belts, new commuters occupy the same amount of urban land as the traditional commuters; the only advantage of the former is more efficient allocation of land between different uses. Also, as new commuters only need to pay $\delta_{s0} < \delta_T$ rather than $\delta_{s0} + \delta_{s1} \equiv \delta_s > \delta_T$ (cf. (3)) per unit of distance to home, their residence would become more peripheral than that of traditional commuters. A lowered cost $\rho$ of an AV, by converting some non-commuters into new commuters, would then increase the city size because the latter require just as much space as the traditional commuters.

It is also possible to modify the model so that residential space enters the utility along with consumption. Then, more centrally located residents, by having to pay higher rents, would occupy less space. If $\delta_{s1}$ is low enough so that AVs return home for day parking, then new commuters live between non-commuters and traditional commuters and thus occupy an intermediate amount of space; lowered $\rho$, by increasing the mass of new commuters at the expense of the other two groups, would have an ambiguous effect on the city size. On the other hand, if $\delta_{s1}$ is high, “parking belt” exists and new commuters have the most peripheral (and thus the largest) residences, increasing mass on new commuters would unambiguously increase the city size.

6. Conclusion

Perhaps the most intriguing finding of this study is that, even in scenarios where the advent of self-driving cars decreases the city size, travel distances increase for at least some groups of commuters. In other scenarios, that are discussed in Section 5 and where cities get bigger, increase in commute distances seems to be a sure thing.

One thing not discussed in the current study is traffic congestion. As all commuter work is more central than non-commuter zone $A_N$, while all commuter residence is more peripheral than $A_N$, all traffic must pass through $A_N$ and that’s where the road capacity is most likely to be binding. If so, the commuter population within a model is essentially fixed, and so
is the non-commuter population, causing the size of zone $A_N$ to be fixed. With cheaper AVs, new commuters cannot be drafted from non-commuters; there is only reallocation from traditional commuters to new ones. Also, if congestion is corrected by a toll, such toll should apply only to AVs loaded with passengers, while empty AVs, optimizing the place to park, should travel free. This somewhat counterintuitive result is due to the fact that empty AVs typically travel in the opposite direction, from center to periphery in the morning, vice versa in the evening; charging them for travel would cause suboptimal (more central) parking, causing problems discussed in the introduction.

References


