A Simple Model of Learning, Brain Drain, Institutional Change, and Return Migration

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Abstract

I propose a simple model of learning centered around the idea that acquisition of a skill is only possible through personal interaction with a person already possessing such skill. In this environment, easy access to credit may lead to "learning bubbles": learning from high-skilled people increases their incomes, which creates the willingness to learn from them even more. The steady state wages of high-skilled people (teachers) are thus very sensitive to the access of low-skilled people (students) to credit. Cross-country differences in the access to credit lead to differences in high-skilled wages, which in turn creates brain drain. Although brain drain reduces the number of high-skilled people in the country, low-skilled people are better off: despite more expensive education, they gain once the education has been acquired. Also, brain drain increases the incentives for the national governments to improve access of low-skilled people to credit. Once the access to credit is improved, there emerges return migration and brain circulation along the transition to new steady state: the arising deficit of skill induces emigrants to return, and some low-skilled people emigrate with an intent to acquire education and return.

1 Introduction

Most of modern labor and development economics literature views human capital as an analog of physical capital. The analogy is so close that in the 1950s and 1960s, when the concept of human capital just emerged, it was accepted with

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hostility by the public, because of direct comparison of humans to machines ((Becker 1992)). The process of human capital accumulation, again by analogy with physical capital accumulation, is associated with utilization of some inputs, usually the time of a student spent on education. Starting with the classic work of (Schultz 1960), all inputs other than the time of the student are viewed as secondary. For example, the article on returns to schooling ((Card 2008)) in the Palgrave Dictionary of Economics does not even mention other inputs.

At the same time, there is an obvious difference between the process of physical capital and human capital accumulation: the latter requires, in most cases, a personal interaction with a person already possessing human capital. As Robert Lucas in his seminal 1988 paper emphasized,

> Human capital accumulation is a *social* activity, involving *groups* of people in a way that has no counterpart in the accumulation of physical capital.

To become a jet pilot, one must interact closely with experienced jet pilots; there is no alternative way of acquiring the skill. For the one who desires to become an expert in the theory of human capital, learning from Gary Becker personally is strictly better than reading Becker's books: personal communication with the author is interactive, while books are mute. Even skills that are labeled by economists as "low" such as taxi driving or hunting in a rainforest requires frequent personal interaction with people already possessing such skills.

Moreover, in post-industrial world learning through personal interaction becomes part of worker's job description throughout most of their careers. For example, software industry changes so rapidly that only those who are very social and who learn permanently may survive. Scientists personally attend conferences on a regular basis, despite unprecedented improvements in communication technologies. Also, in the past several decades Western economies have had an expansion of the service sector; I argue that acquisition of skills in the service sector is different from that in the industrial sector. The latter requires knowledge of math, physics, chemistry, and other well-formulated theories which can be uniformly taught and applied across the globe. Provision of services requires knowledge of consumer psychology which may differ from market to market, and which is hard to standardize in a textbook; learning in the service sector is associated with intense interaction with a live teacher. This matches up with the finding of (Buera and Kaboski 2009) that the share of teaching services in the educational sector rose from 73% in 1950 to 83% in

2000: teachers are becoming more important, relative to other inputs of the educational process, than they were before. Also, the classic assumption that the input of educational process is the opportunity cost of a student originates from half-century old findings of (Schultz 1960); since then, the skill premium (difference between wages of high-skilled and low-skilled) has increased from 1.25 in 1950 to 2.0 in 2000 ((Buera and Kaboski 2009)), which has obviously reduced the opportunity cost of education as measured in high-skilled wages.

In this paper, I propose a simple model of knowledge acquisition. I assume there are two types of individuals, low-skilled (also referred to as *students*) and high-skilled (*teachers*). I will define *current* product of labor as the amount of goods and services that a person can produce without being engaged in teaching or learning.

My model of learning is based on the following formal assumptions:

1. Students can acquire knowledge only through *personal* communication with teachers.
2. Given localized nature of interaction between students and teachers, the former may compensate the latter for receiving education.

The last assumption implies that students earn less than their current product of labor, and those who teach earn more. While the former effect is a very well-known phenomenon in the on-the-job learning literature (see for example (Heckman and Lochner 1998)), the latter effect is novel to my knowledge. The existing literature on human capital accumulation, that assumes student's time as the main input of human capital production and neglects the role of teachers, does not capture this effect.

But this effect is not the end of the story: given that students (low-skilled) earn below their current product of labor and teachers (high-skilled) earn above, the difference in wages is higher than the difference in productivities, which induces students to study even more, and increases the student-teacher wage gap even further. This may lead to formation of a *learning bubble*: as the cost of education rises to infinity, the demand for education does not necessarily decrease to zero. The reason is simple: the cost of education for students is also the income from education for teachers; rising cost of education means rising income of teachers (that is, high-skilled individuals), which means increased willingness of students to acquire skill, which can only be acquired through personal interaction with the existing teachers. Moreover, students' expectations about increasing skill premium in the future will make them study (interact with existing high-skilled) more intensively today, which will increase current income of existing high-skilled people. Thus, expectations about skill premium may be self-fulfilling in this simple model, which makes skill acquisition process a learning bubble.

Much like financial bubbles, learning bubbles are only possible if the costs of transactions are low. In the education industry, transaction costs are manifested in the form of financial constraints: students, which have low skill and low income, have to pay teachers for education; the ability of students to pay depends greatly on their access to credit. Thus, a learning bubble, when the cost of education rises to very high levels, is only possible when students are able to borrow large amounts of money.

Within this framework, we can offer another explanation of the increased skill premium in the past five decades. With improved access of the low-skilled people to credit, they bid more for the same amount of education, which increases the cost of education and consequently incomes of teachers/high-skilled people, which further increases the willingness to study. Thus, with a modest improvement in the access to credit and unchanged current product of labor, the skill premium may considerably increase.

An important result of this modeling framework – that cross-country differences in the access to credit may lead to differences in skill premium – lead us to new insights in the theory of international migration. Suppose there are two countries: "North" with good and "South" with poor financial institutions; then, skill premium is higher in the North. That induces *high*-skilled people from the South emigrate to the North, because of lack of access to credit of *low*-skilled people. The existing literature, to my knowledge, explains international migration of labor by cross-country differences in productivities, or differences in purchasing power of national and foreign currencies, or simply different psychic costs of living at home and abroad. In my theory, differences in the quality of financial institutions are sufficient for international migration to emerge.

Suppose that Southern financial institutions can be improved at some cost; then, I show, payoffs of such improvement increase as the South becomes more open to international migration. Indeed, in a closed economy poor institutions do not prevent human capital accumulation: high-skilled people offer teaching services for free or for a low fee. In an open economy, poor financial institutions cause brain drain; in other words, lowered migration barriers "discipline" governments and induce them to improve access to credit. This result is analogous to an observation that lowered barriers to international trade discipline manufacturers and induce them to increase productivity.

In this paper, I also study the effects of a sudden improvement of the Southern institutions. The long-run effects are trivial, but the study of transition dynamics offers some new insights. An improvement in financial institutions increases the demand for education and thus creates a lack of teachers. High-skilled emigrants return; but if the improvement was sufficiently large, the return of foreign diaspora is not sufficient to cover the deficit of teachers. Assuming that Northerners never migrate to the South (which is not far from reality), there emerges a *brain circulation*: low-skilled Southerners travel to the North to get education, and return once the education has been acquired.

2 Closed Economy

I start with a description of one-country ("closed economy") steady state, and then move on to a two-country setting to model migration between the two countries.

This is a general equilibrium dynamic model. Time is discrete; at each moment of time, there is a continuum of mass M of individuals that are divided into two types – low-skilled and high-skilled. I denote by m the fraction of high-skilled individuals in the economy.[1]. Between any two days, a randomly chosen fraction $1-\delta$ of all individuals (that is, a mass $(1-\delta)M$) dies. The same mass of new individuals is born; therefore, the total population remains constant.[2] Every new individual is assumed to be low-skilled.

There is one consumption good, which is produced using the only input – skill – in a way specified below. All individuals have linear utility: they maximize their discounted stream of consumption. In a benchmark scenario, I assume that borrowing is only limited by the "no-Ponzi-game" restriction,[3] and therefore maximization of discounted stream of consumption is equivalent to maximization of discounted stream of earnings:

$$U_i = \sum_{\tau=\tau_i}^{\infty} \beta^{\tau-\tau_i} y_{i,\tau}$$

[1]For convenience, I denote all exogenously given parameters and functions either by uppercase Latin or lowercase Greek letters; all endogenous variables – by lowercase Latin letters

[2]The model can be easily augmented to the one with positive population growth; a positive growth would reinforce many results in the model

[3]see (Kamihigashi 2008) for a description of Ponzi games

Where i is the index of an individual, τ_i is the birthdate of individual i, τ is the index of time, $y_{i,\tau}$ is the income of individual i at time τ, and β is the discount factor. Given that death occurrence is random, individuals do not know the moment of their death and calculate their utility on an infinite time horizon. I assume that the death possibility is already built into the discount factor (thus $\beta \leq \delta < 1$), and therefore the parameter δ does not explicitly enter the decision-making process.

The only way for low-skilled people (students) to become high-skilled is personal interaction with existing high-skilled people (teachers). The process of learning is stochastic and depends on the *learning intensity* of students $x \geq 0$, which has two possible interpretations. First, it could be viewed as a fraction of time a student has spent learning (within each discrete time period). Alternatively, we can assume that all students learn full time, but in classes of variable size: low intensity x means learning in a large class, while high x implies learning in a small class, or even individually, or even individually with several instructors. Anyway, in this stylized model x must be equal to the ratio of teachers to students in equilibrium; empirically, x can be measured as a ratio of high-skilled to low-skilled people in a group of interest (country, firm, etc.)

If a low-skilled individual learns with intensity x, his probability of becoming high-skilled on a given day is $P(x)$ where $P : R_+ \to [0, 1]$ is a smooth concave non-decreasing function.

2.1 High-skilled individuals

Given that skill is the only state variable in the model, and high skill is the terminal state, there are no intertemporal choices that high-skilled individuals have to make: their problem is essentially static. Each day, high-skilled individuals produce one unit of the consumption good, whose price is normalized to unity, and provide one unit of teaching services. I assume that production and teaching occur simultaneously, and there is no opportunity cost of either of these two services. Therefore, both production and teaching effort are supplied inelastically; in fact, high-skilled people in this stylized model do not have to solve any optimization problem. They earn one unit of income from production, and w_τ units of income from teaching, where w_τ is determined endogenously. Therefore, the value of being high-skilled at time τ_0 can be described as

$$v_h(\tau_0) = \sum_{\tau=\tau_0}^{\infty} \beta^{\tau-\tau_0}(1 + w_\tau) \tag{1}$$

In a steady state where teacher's wage w_τ is constant over time, the above expression simplifies to

$$v_h = \frac{1+w}{1-\beta} \tag{2}$$

where w is the steady-state teacher's wage. Given that most of subsequent analysis is the analysis of steady states, I suppress time indices throughout most of the paper.

2.2 Problem of the low-skilled and demand for education

The productivity of the low-skilled individuals is (normalized to) zero: they cannot produce any consumption good. Thus, there is no opportunity cost of education and the low-skilled individuals spend all their time learning. This assumption contrasts with the assumption made in most of labor economics that the opportunity cost of learning is the major part of educational costs.

As mentioned above, each low-skilled individual (student) can choose the learning intensity x that can be interpreted as the number of teachers per student; the educational expenses are thus $w \times x$. Given this learning intensity, the probability of being high-skilled is $P(x)$; therefore, the value of being low-skilled can be defined as

$$v_l = \max_{x \geq 0} -wx + \beta\left[P(x)v_h + (1 - P(x))v_l\right] \tag{3}$$

Assuming that the economy is in steady state, by manipulating with (2) and (3) we can show that

$$\frac{w}{w+1} = \frac{P'(x)}{\frac{1-\beta}{\beta} + P(x) - P'(x)x} \tag{4}$$

(see appendix A for proof). The left-hand side is increasing with w, while the right-hand side is decreasing in x (the numerator is decreasing, while the denominator is increasing in x); that proves that demand is downward sloping. Another interesting property of demand for education is that under mild restrictions on $P'(0)$, it is bounded away from zero: as the teacher's wage increases to infinity, the demand decreases to a limit which is strictly greater than zero.

Indeed, with infinite wages, the left-hand side of (4) converges to unity, and so should the right-hand side:

$$P'(x) = \frac{1-\beta}{\beta} + P(x) - P'(x)x \tag{5}$$

Assuming that

$$P'(0) > \frac{1-\beta}{\beta} \tag{6}$$

it is straightforward to observe that the left-hand side of (5) is greater than its right-hand size when $x = 0$. The left-hand size is decreasing in x while the right-hand side is increasing; therefore, there exists a unique $\underline{x} > 0$ where they are equal. This $\underline{x}$ is the lower bound of demand for education: demand converges to $\underline{x}$ as w converges to infinity.

This demand is shown on figure 1. For illustration purposes, I differentiate "naive" and "rational" demand. Naive demand does not account for the fact future income of the student increases as educational expenses increase; in other words, it treats the value of being high-skilled v_h as constant and dot dependent on w. Different levels of naive demand correspond to different levels of v_h. This naive demand is consistent with the existing literature on human capital accumulation, which does not associate the costs of education with future incomes of the learners.

"Rational" demand, on the other hand, does account for the fact that increased w changed both the cost of education and expectations of future income: as the wage of teachers increases, students update expectations about their own future income and move to a higher naive demand curve. Equation (4) describes the rational demand curve.

2.3 Supply of teachers and equilibrium

The fraction of high-skilled people in the economy is m; therefore, the teacher-student ratio in the closed economy is $\overline{x} = \frac{m}{1-m}$. The long-run fraction of high-skilled people is determined by the fact that exit of high-skilled due to death must be equal to entry of high-skilled due to learning:

$$(1-\delta)m = \delta P(\overline{x})(1-m) \tag{7}$$

$$\frac{P(\overline{x})}{\overline{x}} = \frac{1-\delta}{\delta} \tag{8}$$

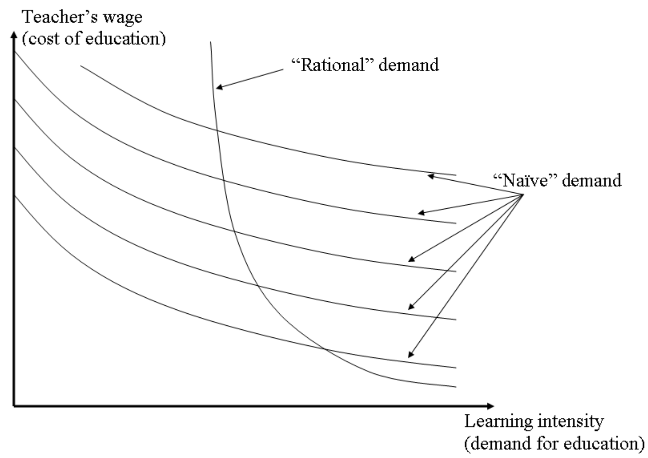

Figure 1: "Naive " and "rational" demand for education

which uniquely determines the long-run supply of teaching services. Note that in closed economy this supply does not depend on wage w, therefore the supply is vertical.

Appendix B shows that steady state equilibrium in the closed economy always exists and is unique. Figure 2 illustrates the steady state.

2.4 Financial constraints

The amount of wealth that students can pass to teachers in exchange for teaching services depends greatly on their ability to borrow. Even in case of informal on-the-job learning, borrowing constraints may be important: although young workers do not have to pay for education they get on the job, they still may need to borrow to smooth their consumption. In the modern world, a housing and a car loan are routine for young professionals; if these loans were not available, young workers could have chosen to learn less and earn more at the beginning of their careers. In the model that I propose, limited access to credit reduces the ability to study, which reduces incomes of the teachers, which reduces the willingness to acquire skill further.

To formalize this idea, I propose a very simple notion of financial constraints. I assume that there exist transaction costs between students and teachers: for a teacher to receive one dollar of teaching income, a student should pay $K \geq 1$

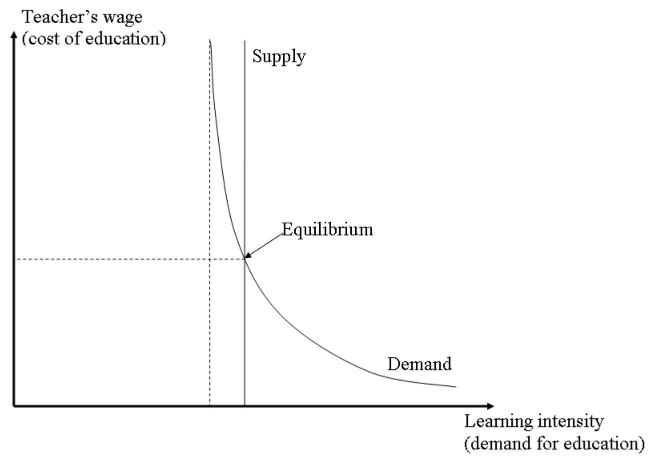

Figure 2: An illustration of the steady state

dollars; the difference of $K-1$ is lost in transaction. A higher K is associated with poorer institutions: it implies a greater waste of resources associated with transactions between teachers and students. This idea of the cost of transactions is analogous to "iceberg" transport costs in international trade literature.

Of course, there exists a more formal way of modeling financial constraints: one could introduce another state variable into the model, savings, and impose a (negative) lower bound of the level of savings. But this would greatly complicate our analysis, and probably not produce any new insights.

With transaction costs, the problem of the low-skilled (3) is redefined as follows:

$$v_l = \max_{x \geq 0} -Kwx + \beta \left[P(x) v_h + (1 - P(x)) v_l \right] \tag{9}$$

It is easy to show that the new demand for education is:

$$\frac{Kw}{w+1} = \frac{P'(x)}{\frac{1-\beta}{\beta} + P(x) - P'(x)x} \tag{10}$$

With increased K, demand for education decreases; the lower bound of the learning intensity $\underline{x}$ also decreases.

Given that the supply of teachers in the closed economy is inelastic, the only effect of higher financial constraints is a decrease in teacher's wages. This finding offers an explanation of the increased skill premium in the past five decades: with the same productivity of skills but with improves access to credit, high-skilled earn more because more people are able to learn from them and better compensate for received education.

3 International migration

3.1 Brain drain

Cross-country differences in institutional quality, manifested in the form of costs of transaction between teachers and students, lead to differences in reward to skill across countries. That creates an incentive for high-skilled people to emigrate from less developed to more developed countries. Unlike the existing literature on international migration which explains migration patterns by differences in productivities, or differences in exchange rates, or "psychic costs" of living at home or abroad, this paper assumes that all these parameters are equal across countries; the only difference among countries is the ease of borrowing for students. If students have limited access to credit, they offer lower rewards to their teachers which makes the latter emigrate; students' access to education thus becomes limited. In this section, I model these considerations more formally.

Suppose there are two countries, North, whose quality of institutions is $K_N = 1$, and South, with $K_S \equiv K > 1$. Thus, South is less developed, with higher costs of transactions. For simplicity, I assume that North is a large country, so its steady state does not depend on international migration; Northern wage of teachers is fixed at $w = w_N$. Southern autarky wage is lower, due to higher costs of transaction, and is equal to $w = w_S^A$. I assume there exist fixed sunk costs of emigration of M, they are sufficiently low so that emigration pays off.

I assume that the number of Southern newly-born people each period is constant and does not depend on migration patters; this assumption allows us to define a steady state with emigration flow.

Given that high-skilled people are homogenous, in the new equilibrium they are indifferent between emigration and non-emigration.[4] Thus, the new Southern wage, w^S is determined from

[4]Corner solutions are impossible: under the assumption that $P'(0) > K\frac{1-\beta}{\beta}$, demand for education in the South is bounded away from zero

$$\frac{1 + w_N}{1 - \beta} - M = \frac{1 + w_S}{1 - \beta} \tag{11}$$

Assuming that M is sufficiently low, we have $w_S > w_S^A$. Higher wage implies lower learning effort (can be computed from (10)), which implies that a smaller fraction of population becomes high-skilled. Moreover, some fraction of the high-skilled emigrates, which reduces the number of high-skilled remaining in the country even further.

More formally, suppose fraction r of high-skilled people emigrates;[5] then, the total number of high-skilled can be computed from

$$(1 - \delta)m = \delta(1 - m)P(x) \tag{12}$$

$$x = \frac{(1 - r)m}{1 - m} \tag{13}$$

The first equation says that the number of Southern high-skilled passing away, at home or abroad, equals the number of newly produced high skilled; the second equation says that the learning intensity must be equal to the ratio of high-skilled remaining in the country to all low-skilled. A reduction of x due to increased wage causes a decrease in m and makes r positive. Figure 3 illustrates the new steady state.

This finding contrasts with much of modern literature on brain drain, which states that learning intensity increases with prospects of emigration; if future emigration is uncertain, some of these extra students will remain in the country (see (Stark, Helmenstein and Prskawetz 1997) for theory and (Chand and Clemens 2008) for recent empirical evidence). This literature, however, does not account for *quality* of education: if the best teachers have left, learning is less efficient even if the same formal degree have been obtained. Moreover, students in developing countries may acquire education not for productive, but for signaling purposes: many recipient countries impose a college degree as one of requirements for immigration. In this model, productive knowledge can be acquired only through personal communication with a teacher; emigration of teachers must reduce the learning intensity.

What are the welfare effects of brain drain in this model? The answer depends on what we understand by "welfare". I suggest the following definition: welfare in this simple economy is the expected lifetime utility of newly-born

[5] all endogenous variable refer to Southern economy henceforth, unless otherwise specified

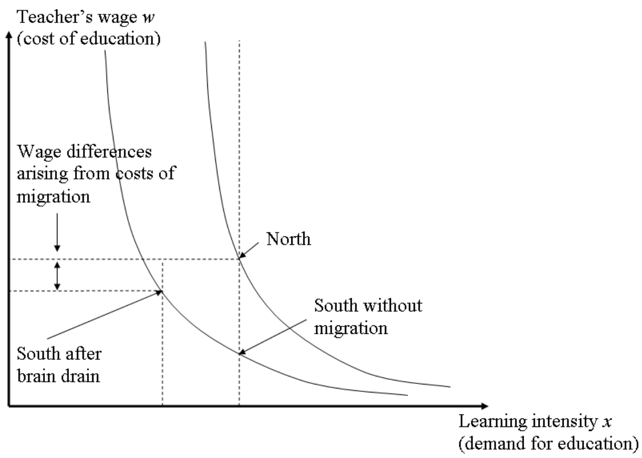

Figure 3: Steady state with brain drain

individuals. Given that all newly-born are low-skilled, the measure of welfare is simply the value of being low-skilled:

$$v_l = \frac{w+1}{1-\beta}\left[\frac{p(x)-p'(x)x}{\frac{1-\beta}{\beta}+p(x)-p'(x)x}\right] \tag{14}$$

With the prospect of emigration, wages of high-skilled individuals increase, which makes the economy move up along the educational demand curve. Numerical examples show that the demand curve is always steeper than isowelfare curves,[6] which means that increased wage of teachers is *increasing* the welfare of students. In other words, unskilled individuals are better off from high-skilled emigration: although this emigration makes education more expensive, they are better off because in the future, they expect to benefit themselves from increased wages and the prospect of emigration. Figure 4 illustrates these results.

3.2 Improvement of institutions

Another result of this model is that increased prospects of emigration increase the incentives of the South to improve its financial institutions.[7] The intuition is as follows. An increased wage w makes demand for education x more re-

[6] formal proof is still underway
[7] Here I offer only a sketch of proof

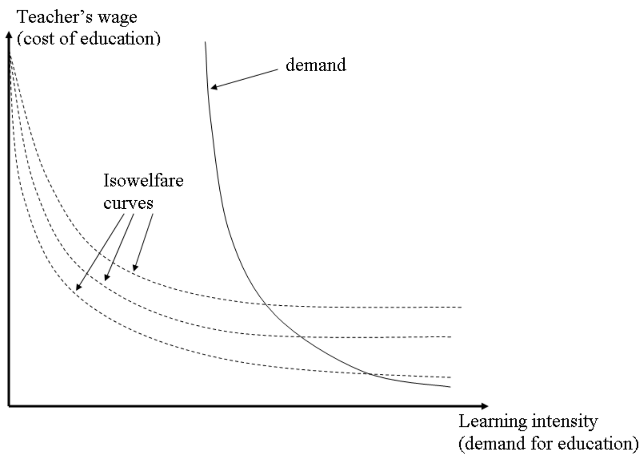

Figure 4: Demand for education vs. isowelfare curves

sponsive to changes in institutions K (which follows directly from (10)). Also, for reasonably high values of x, $\frac{d^2 v_l}{dx^2} < 0$: with higher wages and thus lower x, welfare is more responsive to changes in x. To summarize, higher wages make both $\left|\frac{dx}{dK}\right|$ and $\frac{dv_l}{dx}$ increase, and thus $\frac{dv_l}{dK}$ is higher in absolute value when wages are high.

Therefore, we can conclude that increased prospects of emigration impose a "discipline" on the Southern government: Southern welfare benefits from improved institutions increase, and thus such an improvement becomes more likely.

3.3 Return migration and brain circulation

Theoretical literature on human capital acquisition and return migration is scarce; to my knowledge, the only two theories are (Santos and Postel-Vinay 2003) and (Mayr and Peri 2008). Both of these make an ad-hoc assumption of a skill premium of returnees: people who have lived abroad and returned become more productive than others. The skill premium is applied *only* to returnees; it does not apply to those who never emigrated, or those who emigrated but not returned. In this paper, I model return migration without such an artificial assumption: return migration emerges when financial institutions at home improve.

With better institutions, low-skilled Southerners can borrow (and thus study)

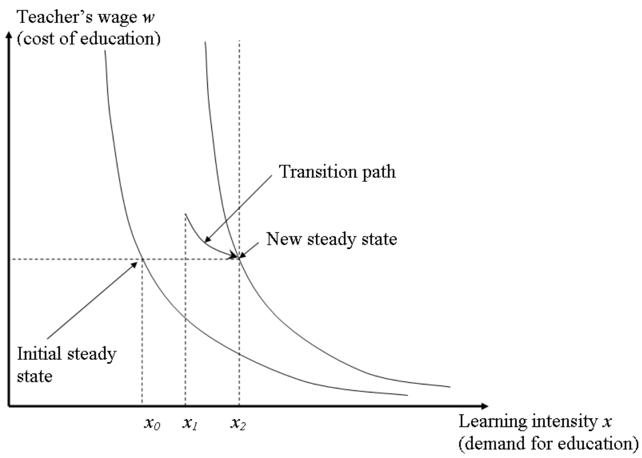

Figure 5: Transition path after institutional improvement. Immediately after improvement, diaspora returns (from x_0 to x_1). Wage soars, until increased demand for skill is satisfied

more, with creates a temporary deficit of skill in the South. Assuming that return of Southerners is free, such a deficit makes Southern emigrants return. Moreover, if the improvement of institutions was substantial, even return of *all* emigrants is not sufficient to cover the deficit. Assuming that Northerners never migrate to the South because of prohibitively high costs of doing so, the economy response to this deficit is as follows. First, the teacher's wages temporarily rise above their new steady state level, and gradually decline as the deficit of skill is filled. Second, after a sufficiently large institutional improvement and sufficiently low emigration costs there may emerge a *brain circulation*: some Southern low-skilled people travel to get education in the North, because the Southern cost of education has temporarily soared above its Northern level. Once the skill has been acquired, the Southerners return for the same reason.

4 Conclusion

In this paper, I build a model of learning which is based on the (rather obvious) assumption that teacher's time is a necessary input for education, and that the only way to become high-skilled is to learn from existing high-skilled people. I show that within such a framework, easy access of students to credit may lead to an emergence of "learning bubbles": learning of low-skilled from the high-skilled raises incomes of the latter, which makes the former even more willing to learn. Expectations about future increase of the reward to skill makes people acquire that skill today, which raises the reward to skill today: expectations are self-fulfilling, which is another property of a bubble.

In this framework, I also show that cross-country differences in the access to credit *alone* may be the driving force of the brain drain: better access to credit of the students means higher reward to skill for the teachers. I also show that emigration prospects improves the welfare of the *low*-skilled people: although brain drain makes the acquisition of skill more difficult, they are better off because of higher incomes once the skill has been acquired.

Lowered emigration costs and resulting increased country openness increases the benefits of institutional improvement: globalization disciplines national governments.

I also show that institutional improvement causes high-skilled emigrants to return. If the improvement was sufficiently large, return of all emigrants is not sufficient to cover the deficit of skill, and there emerges a brain circulation: low-skilled people travel abroad to get education, and return thereafter.

A Derivation of demand for education

The steady-state problem of low-skilled individuals is given by (3)

Under mild restrictions on $P(\cdot)$ specified in (6), the solution of the maximization problem (3) is always interior. Hence, the first-order condition is

$$-w + \beta P'(x)\,(v_h - v_l) = 0 \tag{15}$$

Combining (3) and (15) and solving for v_l, we get

$$(1-\beta)v_l = -w \times x + \frac{P(x)}{P'(x)}w \tag{16}$$

Using (15) again and (2), we can show that

$$\begin{aligned}(1-\beta)v_l &= (1-\beta)\left(v_h - \frac{w}{\beta P'(x)}\right) &(17)\\ &= (w+1) - \frac{1-\beta}{\beta}\frac{w}{P'(x)} &(18)\end{aligned}$$

Combining (16) and (18), we get (4), which implicitly defines the demand for education in a steady state.

B Existence and uniqueness of demand

Demand for education is downward sloping and supply is vertical, therefore the equilibrium, if it exists, is unique. For existence, it is sufficient to show that the lower bound for demand $\underline{x}$ is less than supply $\overline{x}$.

The lower bound for demand is where wages are infinite, and thus $\frac{w}{w+1} = 1$, and thus (from 4)

$$P'(\underline{x}) = \frac{1-\beta}{\beta} + P(\underline{x}) - P'(\underline{x})\underline{x} > \frac{1-\beta}{\beta} \tag{19}$$

This and (8) imply

$$P'(\underline{x}) > \frac{1-\beta}{\beta} > \frac{1-\delta}{\delta} = \frac{P(\overline{x})}{\overline{x}} > P'(\overline{x}) \tag{20}$$

Which means $\underline{x} < \overline{x}$.

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